Effect of Hadronic Rescattering on Elliptic Flow Following Hydrodynamics Model

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The effect of hadronic rescattering on the elliptic flow has been investigated by the Cooper-Frye hadronization model from hydrodynamic evolution following by the afterburner hadronic rescattering model for 200 GeV/c Au + Au at 20-40% centrality. It is found that the hadronic rescattering can suppress elliptic flow v_2 in momentum space especially in lower transverse momentum region. In addition, the hadronic rescattering effects on transverse momentum spectra and anisotropy coordinate space of hadrons are studied.

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I. INTRODUCTION

Lattice QCD calculations have predicted a transition from quark-gluon matter (QGP) to hadronic matter at high temperature or density [1, 2]. This transition was believed to occur around ten microseconds after Big Bang. Practically, one may obtain this piece of information by studying the relativistic heavy ion collisions at Brookhaven National Laboratory now. Some possible probes have been proposed for searching for it [3, 4]. Elliptic flow v_2 is one of these probes, from which one may understand some early information in relativistic heavy ion collision through measuring anisotropy of momentum space of particles in final state. Some experimental results from BNL-RHIC have been published recently [5, 6, 7, 8, 9, 10, 11].

As we know, hydrodynamical model, which is one method to study relativistic heavy ion collisions, can reproduce v_2 data very well below $\sim$ 2 GeV/c of transverse momentum (p_T) [12, 13, 14, 15, 16]. However, the hadronical rescattering effect on elliptic flow after the hadronization following by the hydrodynamics, at least in our knowledge, has not been investigated in details so far. Based on this motivation, we shall focus the influence of rescattering in hadronic state on elliptic flow v_2. It should be pointed that hadronic phase space before rescattering is produced directly by an anisotropic momentum space of produced hadrons in order to produce v_2 in our calculation, which will be described in detail in following. As a result, we find that v_2 of final particles is suppressed due to the rescattering among hadrons to a considerable extent. It may indicate that the effect of rescattering among hadrons on v_2 is not ignored in the research on v_2.

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II. HYDRODYNAMICS MODEL AND INITIAL PHASE SPACE OF HADRONS

Hydrodynamics of relativistic heavy ion collision [17, 18, 19] is defined by (local) energy-momentum and net charge conservation,

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N_i^\mu = 0, \tag{1}$$

where $T^{\mu\nu}$ denotes the energy-momentum tensor and N_i^μ the net four-current of the ith conserved charge. Here will only take the net baryon number. We implicitly assume that all other charges which are conserved on strong-interaction time scale, e.g., strangeness, charm, and electric charge, vanish locally. The corresponding four-currents are therefore identically zero, and the conservation equations are trivial.

For ideal fluids, the energy-momentum tensor and net baryon current assume the simple form:

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p g^{\mu\nu}, \quad N_B^\mu = \rho_B u^\mu, \tag{2}$$

where ϵ, p, and ρ_B are energy density, pressure, and net baryon density in the local rest frame of the fluid, which is defined by $N_B^\mu = (\rho_B, \overrightarrow{0})$, respectively. Here $g^{\mu\nu}$ = diag(+,-,-,-) is the metric tensor, and $u^\mu = \gamma(1, \overrightarrow{v})$ the four-velocity of the fluid ($\overrightarrow{v}$ is the three-velocity and γ = $(1 - \overrightarrow{v}^2)^{-1/2}$, the Lorentz factor).

For simplicity, the hydrodynamics that we are using assumes cylindrically symmetric transverse expansion with a longitudinal scaling flow profile, $v_z = z/t$ though we are investigating the non-central collision. We recognize that our initial phase space of hadrons in this way is not perfect since we put an initial anisotropy of phase space by hand just after Cooper-Frye hadronization as illustrated in the following. Nevertheless, we think that it is still useful and interesting to investigate the hadronic rescattering effect on spectra and flow based on this treatment. In this point, our simulation is somehow like that we start from an assumed initial phase space of hadrons and then study the hadronic rescattering effect. This phase space

is similar to a sudden hadronization phase space which the partonic anisotropy turns to hadron anisotropy. For hydrodynamics, at $z = 0$, Eqs (1) reduces to

$$\partial_t E + \partial_T[(E+p)v_T] = -(\frac{v_T}{r_T} + \frac{1}{t})(E+p), \qquad (3)$$

$$\partial_t M + \partial_T[Mv_T + p] = -(\frac{v_T}{r_T} + \frac{1}{t})M, \qquad (4)$$

$$\partial_t R + \partial_T[Rv_T] = -(\frac{v_T}{r_T} + \frac{1}{t})R, \qquad (5)$$

where we defined E $\equiv T^{00}$, M $\equiv T^{0T}$, and R $\equiv N^0_B$. In the above expressions, the index T refers to the transverse component of the corresponding quantity. The equations (3)-(5) describe the evolution in the $z = 0$ plane. Because of the assumption of longitudinal scaling, the solution at any other $z \neq 0$ can be simply obtained by a Lorentz boost.

Initial conditions for Au + Au at 200 GeV/c in 20-40% centrality is supposed that initial energy density = 10.3 GeV/fm^3 and initial time = 0.6 fm/c, and a hydrodynamical evolution of cylindrically symmetric transverse expansion with a longitudinal scaling flow profile is adopted. When temperature of liquid reaches to a critical energy density (= 0.42 GeV/fm^3), a hadronization of QGP which is described by employing the MIT bag model EOS [21] will take place. And an ideal hadron gas is established and the corresponding EOS is applied. Hadronization in hydrodynamics usually takes the method of Cooper and Frye [22]. In this work, however, we make an assumption that the coordinate and momentum spaces of hadronized particles are not isotropic but have shapes as plotted in Figure 1 in order to simulate Au + Au at 200 GeV/c in 20-40% centrality and get v_2 in our calculation. Here the mean anisotropy in momentum space, $F \equiv$<Px/Py>, takes 1.18 in our simulation for all produced hadrons.

For the details of the simulation to result in the above anisotropic distributions, we wrote

$$R_x = R_{Tx}cos\theta, \quad P_x = P_{Tx}\cos\psi$$

$$R_y = R_{Ty}sin\theta, \quad P_y = P_{Ty}\sin\psi$$

$$R_z = t * tanhy, \quad P_z = m_T \sinh y$$

$$t = \xi tanhy, \quad E = \sqrt{P_x^2 + P_y^2 + P_z^2 + m_0^2} \qquad (6)$$

$$< P_{Tx}/P_{Ty} >=< R_{Ty}/R_{Tx} >= F \qquad (7)$$

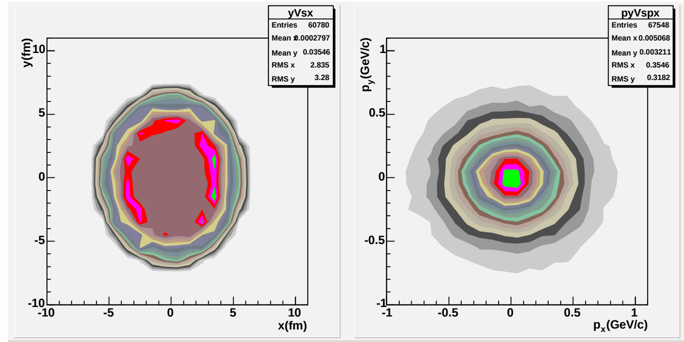

FIG. 1: Contour plots of shapes of the phase space where hadrons are produced when two same spherical overlap in half-central collision. Left: transverse plane at z = 0 in coordinate space; Right: transverse plane at Pz = 0 in momentum space.

$$< \theta/\psi >= 1 \qquad (8)$$

where $m_T = \sqrt{p_T^2 + m_0^2}$, (R_x, R_y, R_z, t) and (P_x, P_y, P_z, E) are the 4-vectors of produced hadrons in coordinate and momentum spaces respectively. We assume that hadrons are randomly distributed in the overlap coordinate space of two spherical nucleus in terms of the left column of equations (7) by hadronization. ξ and y are random numbers which are respectively randomly distributed in [0,14] fm/c and [-5,5]. As for momentum and position of hadrons, P_{Tx} and R_{Ty} denote a random variable distributed in [0,4] GeV and [0,7] fm, then P_{Ty} and R_{Tx} can be obtained by the anisotropic factor in momentum and coordinate space, namely F, which makes our momentum and coordinate space distribution in transverse plane elliptic and anti-elliptic. If we make F equal to 1, our hadronization method will go back to the method of Cooper and Frye. θ and ψ are the azimuths distributed in [0,2π] in coordinate and momentum spaces. But here we assume these produced hadrons are from a radial source in which θ and ψ have a correlation like the equation (8), which means that once a ψ has been decided, the corresponding θ will be selected in a Gaussian distribution whose mean is ψ and whose width is always assumed to be 0.2π in our calculation. The set of equations results in the coordinate and momentum space as figure 1 shows. Here we choose the anisotropic factor F in momentum space, equals to 1.18 for 200 GeV/c Au + Au in 20-40% centrality.

III. HADRONIC RESCATTERING MODEL

After the hadrons are produced, they enter next rescattering process among them. We here use the rescattering model from LUCIAE model [23, 24]. Two particles will

collide if their minimum distance d_{min} satisfies

$$d_{min} \leq \sqrt{\frac{\sigma_{total}}{\pi}}, \tag{9}$$

where σ_{total} is the total cross section in fm^2 and the minimum distance is calculated in the C.M.S. frame of the two colliding particles. If these two particles are moving towards each other at the time when both of them are formed, the minimum distance is defined as the distance perpendicular to the momentum of both particles. If the two particles are moving back-to-back, the minimum distance is defined as the distance at the moment when both of them are formed. Assuming that the hadrons move along straight-line classical trajectories between two consecutive collisions it is possible to calculate the collision time when two hadrons reach their minimum distance and order all the possible collision pairs according to the collision time sequence. If the total and the elastic cross section satisfies

$$\frac{\sigma_{elastic}}{\sigma_{total}} \geq \eta \tag{10}$$

where η is a random number then the particles will be elastically scattered or else the collision will be considered as an inelastic collision. The distribution of the momentum transfer, t, is taken as

$$\frac{d\sigma}{dt} \sim exp(Bt) \tag{11}$$

where B, for an elastic scattering, depends on the masses of two scattering particles. The azimuthal angle will be isotropically distributed.

During the rescattering process, the following inelastic reactions are included in our calculation:

πN $\rightleftharpoons \Delta\ \pi$	πN $\rightleftharpoons \rho$ N
N N $\rightleftharpoons \Delta$ N	$\pi\ \pi \rightleftharpoons$k $\overline{k}$
π N $\rightleftharpoons$ k Y	$\pi\ \overline{N} \rightleftharpoons \overline{k}\ \overline{Y}$
π Y $\rightleftharpoons$ k Ξ	$\pi\ \overline{Y} \rightleftharpoons \overline{k}\ \overline{\Xi}$
$\overline{k}$ N $\rightleftharpoons \pi$ Y	k $\overline{N} \rightleftharpoons \pi\ \overline{Y}$
$\overline{k}$ Y $\rightleftharpoons \pi\ \Xi$	k $\overline{Y} \rightleftharpoons \pi\ \overline{\Xi}$
$\overline{k}$ N $\rightleftharpoons$ k Ξ	k $\overline{N} \rightleftharpoons \overline{k}\ \overline{\Xi}$
$\pi\ \Xi \rightleftharpoons$ k Ω^-	$\pi\ \overline{\Xi} \rightleftharpoons \overline{k}\ \overline{\Omega^-}$
k $\overline{\Xi} \rightleftharpoons \pi\ \overline{\Omega^-}$	$\overline{k}\ \Xi \rightleftharpoons \pi\ \Omega^-$
$\overline{N}$ N annihilation	$\overline{Y}$ N annihilation

where the hyperons $Y = \Lambda$ or Σ. The relative probabilities for the different channels, e.g. in (π N)-scattering, is used to determine the outcome of the inelastic encounter. As the reactions introduced above do not make up the full inelastic cross section, the remainder is again treated as elastic encounters.

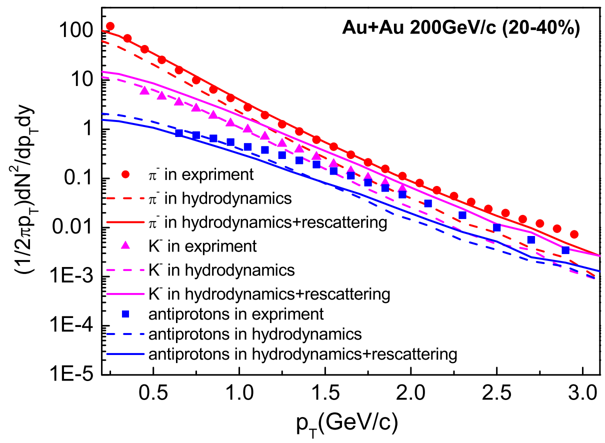

FIG. 2: The transverse momentum distributions of π^-, k^-, $\overline{P}$ in Au + Au at 200GeV/c in 20-40% centrality in our hydrodynamical + rescattering model. Full lines and dash lines represent p_T spectra after and before hadronic rescattering, respectively; solid symbols are experimental data which come from [25].

IV. RESULTS AND DISCUSSIONS

Figure 2 shows the transverse momentum distribution of π^-, k^-, $\overline{P}$ before and after rescattering in Au + Au at 200GeV/c in 20-40% centrality in our hydrodynamical calculation followed by the rescattering model.

At the same time, we obtain the v_2 of hadrons before and after rescattering in terms of formula (12), which indicates the anisotropy of hadrons' momentum space.

$$v_2 \equiv < \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} > \tag{12}$$

Figure 3 shows dependence of v_2 of π^-+k^-, h^-+h^+, $\overline{P}$, Λ+$\overline{\Lambda}$ on p_T before and after rescattering in hydrodynamics and experimental data in Au+Au at 200GeV/c in 20-40% centrality [25].

In order to study anisotropy of hadrons' coordinate space, the parameter named as ϵ_2 is defined by formula (13).

$$\epsilon_2 \equiv < \frac{x^2 - y^2}{x^2 + y^2} > \tag{13}$$

The plots (a) (c) and (e) in figure 4 give p_T spectra, v_2 and ϵ_2 of four types of particles ((π^++π^-), (k^++k^-), (P+$\overline{P}$) and (Λ+$\overline{\Lambda}$)) vs p_T before and after rescattering in hydrodynamics+rescattering model. The open and filled points, respectively, stand for the status before and after hadrons' rescattering. We can see that p_T yield and ϵ_2 of hadrons is enhanced by rescattering, and also note ϵ_2 changes its sign through rescattering. On the other hand, elliptic flow v_2 of hadrons is suppressed by the rescattering. In order to quantify the extent of enhancement (or

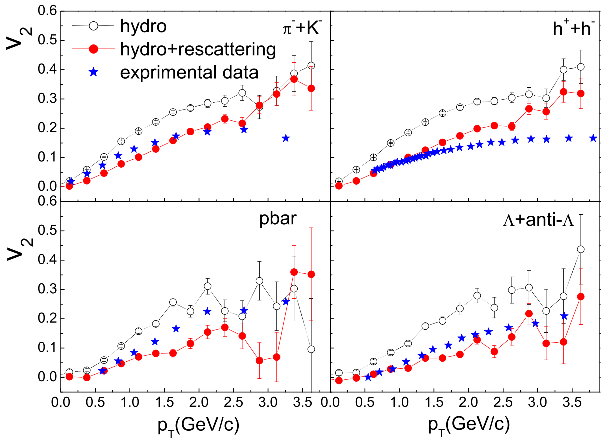

FIG. 3: The p_T dependences of v_2 of π^-+k^-, h^-+h^+, $\overline{P}$ and $\Lambda + \overline{\Lambda}$ before and after rescattering in hydrodynamical+rescattering model and experimental data which come from [5] and [11] in Au+Au at 200 GeV/c in 20-40% centrality.

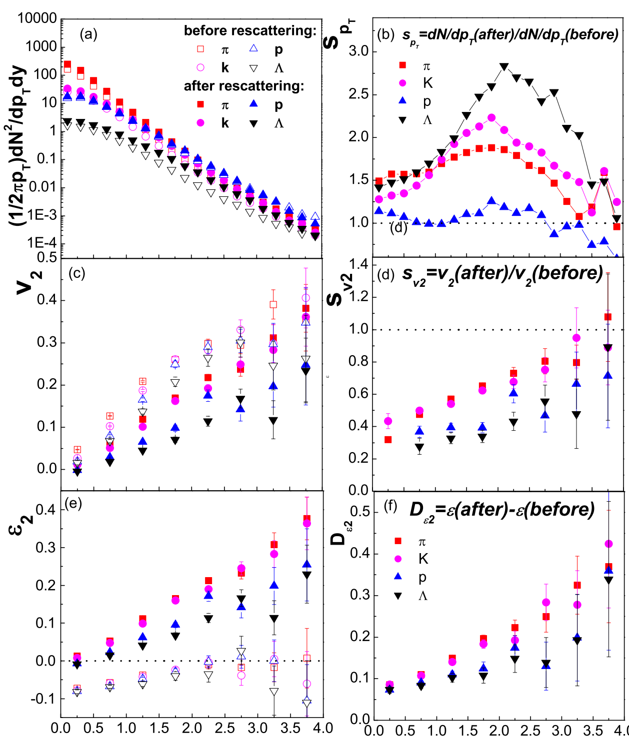

FIG. 4: (a),(c) and (e): The p_T dependences of spectra, v_2 and ϵ_2 of $\pi^++\pi^-$ (square), k^++k^- (circle), P+$\overline{P}$ (up-triangle) and $\Lambda+\overline{\Lambda}$ (down-triangle) before (open symbols) and after (full symbols) rescattering in hydrodynamical + rescattering model in Au+Au at 200 GeV/c in 20-40% centrality; (b),(d) and(f): The p_T dependences of s_{P_T}, s_{v_2} and D_{ϵ_2}. See more details in text.

suppression) at different p_T bins, we define three factors named as s_{P_T}, s_{v_2} and D_{ϵ_2} by formula (14),(15) and (16).

$$s_{P_T}(p_T) \equiv \frac{\frac{dN}{dydP_T}_{after}}{\frac{dN}{dydP_T}_{before}} \tag{14}$$

$$s_{v_2}(p_T) \equiv \frac{V_2(p_T)_{after}}{V_2(p_T)_{before}} \tag{15}$$

$$D_{\epsilon_2}(p_T) \equiv \epsilon(p_T)_{after} - \epsilon(p_T)_{before} \tag{16}$$

The plots (b) (d) and (f) in figure 4 show respective dependence of s_{P_T} s_{v_2} and D_{ϵ_2}on p_T. In (b), we find that the yield of hadrons is increased by hadrons' rescattering, especially for higher p_T yield. It indicates the rescattering produced many secondary hadrons,whose p_T seems harder than those hadrons in last generation. In (d), s_{v_2} increases with p_T, which indicates that the suppression effect on v_2 from hadrons' rescattering becomes weaker and weaker with the increasing of p_T. In (e) and (f), we see the rescattering turns the shape of coordinate space, and the effect becomes stronger with the increasing of p_T. It indicates the rescattering makes hadrons' momentum space less anisotropy and more heavier particles, more suppression the v_2. While the changes of the tropism of coordinate space occurs in the same time. In our calculation, at least 20 $\sim$ 40% v_2 will disappear through final hadrons' rescattering. So the effect from final hadrons' rescattering on v_2 maybe should be considered in order to research early state information in relativistic heavy ion collisions.

In order to investigate the initial geometrical dependence of our results farther, we next attempt to take Cooper-Frye hadronization method but with three different anisotropic conditions to see the change of effect from final hadrons' rescattering. Since the status of momentum and coordinate space before the hadrons' rescattering can not be measured by us, we assume anther three conditions here.

(I) We decrease the correlation between momentum space and coordinate space, i.e. increase σ= 0.2π to σ = 0.5π. As we see the circles and squares in figure 5, the stronger correlation seems to produce more hadrons and suppress v_2 more strongly, but its change for the anisotropy of coordinate space ϵ_2 is slight.

(II) We only remain anisotropy of coordinate space when hadronization takes place. What will happen after rescattering among hadrons? The diamons in figure 5 show our results. Obviously, only hadronic rescatterring can not induce v_2 if the interaction system has not anisotropy of momentum space before the hadronic rescattering, though its high s_{P_T} indicates that more secondary hadrons are produced in rescattering. On the other hand, the anisotropy of coordinate space also disappear after hadrons' rescattering. It is also reasonable

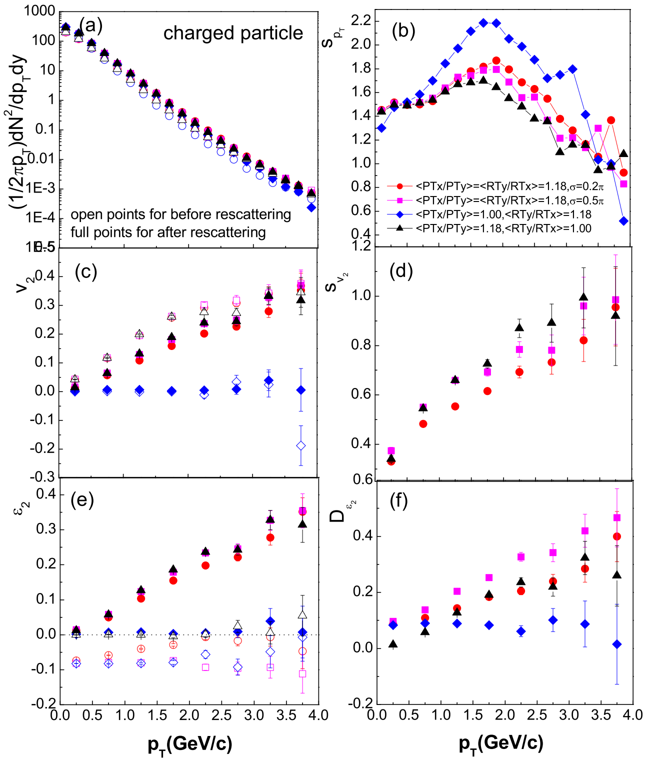

FIG. 5: (a), (c) and (e): The p_T dependences of spectra, v_2 and ϵ_2 of charged particle before (open symbols) and after (full symbols) rescattering in hydrodynamical + rescattering model in Au + Au at 200 GeV/c in 20-40% centrality. The circles and squares are for the hadronization with both initial v_2 and ϵ_2, and the circles correspond to 0.2π of the width of ψ-ϕ correlation and squares correspond to 0.5π. The diamonds are from the hadronization whose particles are only with initial $\epsilon_2$and the triangles are from the hadronization whose particles are only with initial v_2 (see the insert of (b)); (b),(d) and (f): The p_T dependences of s_{P_T}, s_{v_2} and D_{ϵ_2} in these four hadronaztion situations.

because the rescattering plays a role that it decreases v_2 and tends to make the reaction system isotropic from above, which also is consistent with the isotropic feature of rescattering angle in LUCIAE rescattering model. So the anisotropy of momentum space, i.e. v_2, may be produced in partonic state before hadronization if the hadronic rescattering has no direction-sense. In other words, flow must be formed before the hadronization.

(III) We only remain initial anisotropy of momentum space but lack initial anisotropy of coordinate space when hadronization takes place as shown with triangles in figure 5, the rescattering still decrease hadrons' v_2 and creat a big anisotropy of coordinate space ϵ_2.

Not only in above three conditions but also in our default condition, we all find that hadrons' rescattering produces many secondary hadrons and suppresses the elliptic flow v_2 of hadrons to a certain extent. When the two anisotropy of coordinate and momentum space tend to be homologic with time, the rescattering ends and the reaction system freezes out.

However, we note that the effect of rescattering on v_2 in high p_T domain is not perfect from Figure 3, i.e., v_2 does not overlap with experimental data which saturates above $P_T \sim$2 GeV/c. We think that one cause may be from our anisotropic factor in the momentum space. Even though the experimental data can be fitted well in lower p_T, it is a bit artificial since the nucleon distribution of transverse plane (x,y) and anisotropic hydrodynamical evolution are not taken into account in this hydrodynamics [13]. The other cause could be the partonic effect on flow, such as partons cascade [26] and jet quenching [27, 28], is also very important for the character of particles with high p_T, which we also do not take into account. Of course, the main goal of this work is to investigate the hadronic rescattering effect on the final state hadronic elliptic flow.

V. CONCLUSIONS

In conclusion, we apply hydrodynamics model followed by the rescattering model to investigate the effect of hadronic rescattering on elliptic flow v_2 for 200 GeV/c Au + Au at 20-40% centrality. Hadronization is treated by Cooper-Frye method and an initial anisotropic phase space of hadrons is assumed. We find that the yield of hadrons is increased after the rescattering. While, the rescattering among the hadrons plays a suppression role for v_2, which makes an asymmetric system in momentum space less anisotropic. The suppression becomes weaker with the increasing of transverse momentum. We miss at least 20 $\sim$ 40% v_2 after hadronic rescattering in our hydrodynamics + rescattering model. This may, however, change if hadronization occurs when partons make their last scattering as treated in Texas multi-phase transport model when the spatial anisotropic is small [29, 30] instead of when the energy reaches certain critical energy density presented here. On the other hand, the hadronic rescattering makes the coordinate space of hadrons tend to the similar directions to the momentum space. In our model, we also find that the rescattering does not induce the elliptic flow if only initial anisotropy of coordinate space exists in the hadronic phase.

Considering that the hadronic system created in RHIC collision is very dense, the following hadronic rescattering may happen potentially and play an important role. Of course, only information of the momentum space after the freeze-out can be measured in experiment. But anyway, the effect of elliptic flow v_2 from final hadronic rescattering deserve attention if the hadronization takes place when the energy reaches certain critical energy density presented here in hydrodynamics.

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